**Question 1: A customer owing a Maruti car right now has got the option to switch over to Maruti Ambassador or Fiat next time with the probability of Vi = (0.2, 0.5, 0.3). Given the transition matrix. Find the probabilities with his fourth purchase.  
  
Answer:** Given that Vi = [0.2 0.5 0.3] and

P = 0.4 0.3 0.3  
 0.2 0.5 0.3  
 0.25 0.25 0.5

Calculating P^2:

P = 0.4 0.3 0.3 0.4 0.3 0.3  
 0.2 0.5 0.3 \* 0.2 0.5 0.3  
 0.25 0.25 0.5 0.25 0.25 0.5  
  
Calculating for individual values for matrix multiplication, we get:

P11 = (0.4 \* 0.4) + (0.3 \* 0.2) + (0.3 \* 0.25) = (0.16) + (0.06) + (0.075) = 0.295  
P12 = (0.4 \* 0.3) + (0.3 \* 0.5) + (0.3 \* 0.25) = (0.12) + (0.15) + (0.075) = 0.345  
P13 = (0.4 \* 0.3) + (0.3 \* 0.3) + (0.3 \* 0.5) = (0.12) + (0.09) + (0.15) = 0.36

P21 = (0.2 \* 0.4) + (0.5 \* 0.2) + (0.3 \* 0.25) = (0.08) + (0.1) + (0.075) = 0.255  
P22 = (0.2 \* 0.3) + (0.5 \* 0.5) + (0.3 \* 0.5) = (0.06) + (0.25) + (0.075) = 0.385  
P23 = (0.2 \* 0.3) + (0.5 \* 0.3) + (0.3 \* 0.5) = (0.06) + (0.15) + (0.15) = 0.36  
  
P31 = (0.25 \* 0.4) + (0.25 \* 0.2) + (0.5 \* 0.25) = (0.1) + (0.05) + (0.125) = 0.275   
P32 = (0.25 \* 0.3) + (0.25 \* 0.5) + (0.5 \* 0.25) = (0.75) + (0.125) + (0.125) = 0.325   
P33 = (0.25 \* 0.3) + (0.25 \* 0.3) + (0.5 \* 0.5) = (0.075) + (0.075) + (0.25) = 0.4  
  
Now calculating P^3  
0.295 0.345 0.36 0.4 0.3 0.3  
0.255 0.385 0.36 \* 0.2 0.5 0.3  
0.275 0.325 0.4 0.25 0.25 0.5  
  
Calculating for individual values for matrix multiplication, we get:  
  
P11 = (0.295 \* 0.4) + (0.345 \* 0.2) + (0.36 \* 0.25) = (0.413) + (0.069) + (0.09) = 0.572  
P12 = (0.295 \* 0.3) + (0.345 \* 0.5) + (0.36 \* 0.25) = (0.0885) + (0.1725) + (0.09) = 0.351  
P13 = (0.295 \* 0.3) + (0.345 \* 0.3) + (0.36 \* 0.5) = (0.0885) + (0.1035) + (0.18) = 0.372  
  
P21 = (0.255 \* 0.4) + (0.385 \* 0.2) + (0.36 \* 0.25) = (0.102) + (0.077) + (0.09) = 0.269  
P22 = (0.255 \* 0.3) + (0.385 \* 0.5) + (0.36 \* 0.25) = (0.0765) + (0.1925) + (0.09) = 0.359  
P23 = (0.255 \* 0.3) + (0.385 \* 0.3) + (0.36 \* 0.5) = (0.0765) + (0.1155) + (0.18) = 0.372  
  
P31 = (0.275 \* 0.4) + (0.325 \* 0.2) + (0.4 \* 0.25) = (0.11) + (0.065) + (0.1) = 0.275  
P32 = (0.275 \* 0.3) + (0.325 \* 0.5) + (0.4 \* 0.25) = (0.0825) + (0.1625) + (0.1) = 0.345  
P33 = (0.275 \* 0.3) + (0.325 \* 0.3) + (0.4 \* 0.5) = (0.0825) + (0.0975) + (0.2) = 0.38

Now calculating Vi^4 = Vi \* P^3:

0.572 0.351 0.372

0.2 0.5 0.3 \* 0.269 0.359 0.372  
 0.275 0.345 0.38

Calculating for individual values for matrix multiplication, we get:

P11 = (0.2 \* 0.572) + (0.5 \* 0.269) + (0.3 \* 0.275) = (0.1144) + (0.1345) + (0.825) = 0.3314

P12 = (0.2 \* 0.351) + (0.5 \* 0.359) + (0.3 \* 0.345) = (0.0702) + (0.1795) + (0.1035) = 0.3532

P13 = (0.2 \* 0.372) + (0.5 \* 0.372) + (0.3 \* 0.38) = (0.0744) + (0.186) + (0.114). = 0.3744

Hence the resultant matrix is [ 0.3314 0.3532 0.3744]. This matrix shows the probabilities.

**Question 2: The transition probability matrix of a Markov chain {Xn}, n= 1, 2, having 3 states 1,2 and 3 is given by a matrix. And the initial distribution of P0 is [0.7 0.2 0.1]**

**Answer 2 (i):** Calculating P(X2 = 3)

Calculating P^2:  
0.1 0.5 0.4 0.1 0.5 0.4

0.6 0.2 0.2 \* 0.6 0.2 0.2

0.3 0.4 0.3 0.3 0.4 0.3

Calculating for individual values for matrix multiplication, we get:

P11 = (0.1 \* 0.1) + (0.5 \* 0.6) + (0.4 \* 0.3) = (0.01) + (0.3) + (0.12) = 0.43

P12 = (0.1 \* 0.5) + (0.5 \* 0.2) + (0.4 \* 0.4) = (0.05) + (0.1) + (0.16) = 0.31

P13 = (0.1 \* 0.4) + (0.5 \* 0.2) + (0.4 \* 0.3) = (0.04) + (0.1) + (0.12) = 0.26

P21 = (0.6 \* 0.1) + (0.2 \* 0.6) + (0.2 \* 0.3) = (0.06) + (0.12) + (0.06) = 0.24

P22 = (0.6 \* 0.5) + (0.2 \* 0.2) + (0.2 \* 0.4) = (0.3) + (0.04) + (0.08) = 0.42

P23 = (0.6 \* 0.4) + (0.2 \* 0.2) + (0.2 \* 0.3) = (0.24) + (0.04) + (0.06) = 0.34

P31 = (0.3 \* 0.1) + (0.4 \* 0.6) + (0.3 \* 0.3) = (0.03) + (0.24) + (0.09) = 0.36

P32 = (0.3 \* 0.5) + (0.4 \* 0.2) + (0.3 \* 0.4) = (0.15) + (0.08) + (0.12) = 0.35

P33 = (0.3 \* 0.4) + (0.4 \* 0.2) + (0.3 \* 0.3) = (0.12) + (0.08) + (0.09) = 0.29

Now calculating:

0.43 0.31 0.26

0.7 0.2 0.1 \* 0.24 0.42 0.34

0.36 0.35 0.29

Calculating for individual values for matrix multiplication, we get:

P11 = (0.7 \* 0.43) + (0.2 \* 0.24) + (0.1 \* 0.36) = (0.301) + (0.048) + (0.036) = 0.385

P12 = (0.7 \* 0.31) + (0.2 \* 0.42) + (0.1 \* 0.35) = (0.217) + (0.084) + (0.035) = 0.336

P13 = (0.7 \* 0.26) + (0.2 \* 0.34) + (0.1 \* 0.29) = (0.182) + (0.068) + (0.029) = 0.279

Hence the resultant matrix is [0.385 0.336 0.279].  
P(X2 = 3) is 0.279.

**Answer 2 (ii):** P0 = [0.7 0.2 0.1]

Hence:

P(X0 = 1) = 0.7

P(X0 = 2) = 0.2

P(X0 = 3) = 0.1

And the transition matrix is given by:

0.1 0.5 0.4

0.6 0.2 0.2

0.3 0.4 0.3

P11 = 0.1 P12 = 0.5 P13 = 0.4

P21 = 0.6 P22 = 0.2 P23 = 0.2

P31 = 0.3 P32 = 0.4 P33 = 0.3

Calculating for P(X3 = 2, X2 = 3, X1 = 3, X0 = 2)

P(X3 = 2, X2 = 3, X1 = 3, X0 = 2)

= P(X3 = 2 | X2 = 3) . P(X2 = 3 | X1 = 3) . P(X1 = 3 | X0 = 2) . P(X0 = 2)

= P32 . P33 . P23 . P(X0 = 2)

= 0.4 \* 0.3 \* 0.2 \* 0.2

= 0.0048

**Question 3: Five green balls and 3 white balls are placed in two boxes A and B so that each box contains 4 balls. At each stage, a ball is drawn at random from each box and the two balls are interchanged. Let X(n) denote the number of white balls in box A after the nth draw.**

**Determine the state space and index set.**

**If Y(n) is the total number of green balls in box A after the nth draw, then determine the state space and index set for Y(n)**

**Answer :**

**State Space and Index Set for X(n)**

State Space for X(n)

The variable X(n) represents the number of white balls in the box A after n draws. Initially box A can contain 0, 1, 2, 3 or 4 white balls, so the state space for X(n) is:

SX(n) = {0, 1, 2, 3, 4}

Index Set for X(n)

The index set refers to the set of all possible values of n. Since n represents the number of draws, n can be any non-negative integer:

TX(n) = {0, 1, 2, 3, …}

**State Space and Index Set for Y(n)**

State Space for X(n)

The variable Y(n) represents the number of green balls in box A after n draws. Initially box A can contain 0, 1, 2, 3, or 4 green balls. Hence the state space for Y(n) is:

SY(n) = {0, 1, 2, 3, 4}

Index Set for Y(n)

The index set refers to the set of all possible values of n. Since n represents the number of draws, n can be any non-negative integer:

TY(n) = {0, 1, 2, 3, …}